## Fourth Semester B.E. Degree Examination, July/August 2021 Design and Analysis of Algorithms

Time: 3 hrs.

## Note: Answer any FIVE full questions.

1 a. Define asymptotic notations with example.
(09 Marks)
b. Solve the following recurrence relation. Using backward substitution :
$\mathrm{x}(\mathrm{n})=3 \mathrm{x}(\mathrm{n}-1)$ for $\mathrm{n}>1, \mathrm{x}(1)=4$
(03 Marks)
c. List and explain the basic asymptotic efficiency classes.
(08 Marks)
2 a. Define the following terms
(i) Graph
(ii) Tree
(iii) Set and Dictionaries.
(04 Marks)
b. Write an algorithm to find $\mathrm{n}^{\text {th }}$ Fibonacci number recursively. Set up a recurrence relation for Fibonacci number and solve it.
(08 Marks)
c. Consider the following algorithm :

Algorithm Mystery (n)
//Input : A nonnegative integer is
$\mathrm{S} \leftarrow 0$
for $\mathrm{i} \leftarrow 1$ to n do
$\mathrm{S} \leftarrow \mathrm{S}+\mathrm{i} * \mathrm{i}$
return s
(i) What does this algorithm compute?
(ii) What is its basic operation?
(iii) How many time is the basic operation executed?
(iv) What is the efficiency class of this algorithm?
(08 Marks)
3 a. Write an algorithm to finding the maximum and minimum of the given set of elements, \{a(i), a(i+1) $\qquad$ .a(j) \}
(08 Marks)
b. Apply Quicksort algorithm to the following set of input values and draw a tree of recursive calls to quicksort with input values $l$ and r of subarray bounds and split position P of a partition obtained.
5,3, 1, 9, 8, 2, 4, 7
(12 Marks)
4 a. Explain the Strassen's matrix multiplication algorithm to compute the product of $2 \times 2$ matrices.
(08 Marks)
b. Describe the advantages and disadvantages of divide and conquer technique. (06 Marks)
c. Consider the following graph, apply the DFS-based algorithm to solve the topological sorting problem for the given digraphs : (Refer Fig. Q4 (c))
(06 Marks)


Fig. Q4 (c)
1 of 3

5 a. Write an algorithm of greedy method control abstraction for the subset paradigm. (06 Marks)
b. What is spanning tree? Explain the Prim's algorithm for constructing a minimum spanning tree for the weighted connected graph.
(08 Marks)
c. Apply the dijkstra's algorithm for single source shortest paths for the given graph and assume vertex 'A' as source (Fig. Q5 (c))
(06 Marks)


6 a. (i) Construct a Huffman code for the following data:

| Character | A | B | C | D | - |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.4 | 0.1 | 0.2 | 0.15 | 0.15 |

(ii) Encode the text ABACABAD using the code of Q(i).
(iii) Decode the text whose encoding is 100010111001010 in the code of $\mathrm{Q}(\mathrm{i})$.
(10 Marks)
b. Construct a heap for the list $2,9,7,6,5,8$ by bottom up algorithm and how efficient is this algorithm in the worst case?
(10 Marks)
7 a. Apply the dynamic programming algorithm for constructing an optimal binary search-tree for the following data set:

| Key | A | B | C. | D |
| :--- | :--- | :--- | :--- | :--- |
| Probability | 0.1 | 0.2 | 0.4 | 0.3 |

b. Solve the all pairs shortest path problem for the diagram with the following weight matrix:
$\left[\begin{array}{ccccc}0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0\end{array}\right]$
(10 Marks)

8 a. Compute the optimal tour of the given directed graph using dynamic programming techniques of TSP. (Refer Fig. Q8 (a)).
(10 Marks)


Fig. Q8 (a)
2 of 3
b. Apply the bottom-up dynamic programming algorithm to the following instance of the knapsack problem.

| Item | Weight | Value |
| :---: | :---: | :---: |
| 1 | 2 | $\$ 12$ |
| 2 | 1 | $\$ 10$ |
| 3 | 3 | $\$ 20$ |
| 4 | 2 | $\$ 15$ |

9 a. Explain how the board's symmetry can be used to find the second solution to the 4 -Queen problems.
(06 Marks)
b. Apply backtracking to the problem of finding a Hamiltonian circuit in the following graph (Fig. Q9 (b))
(08 Marks)


Fig. Q9 (b)
c. Write a pseudocode of the backtracking algorithm.
(06 Marks)
10 a. Construct and draw the state space tree of the backtracking algorithm applied to the instance $A=\{3,5,6,7\}$ and $d=15$ of the subset problem.
(10 Marks)
b. Solve the following instance of the knapsack problem by FIFOBB algorithm.

$$
\begin{array}{ll}
\mathrm{n}=4 & \left(\mathrm{P}_{1}, \mathrm{P}_{2}, \mathrm{P}_{3}, \mathrm{P}_{4}\right)=(10,10,12,18) \\
& \mathrm{W}_{1}, \mathrm{~W}_{2}, \mathrm{~W}_{3}, \mathrm{~W}_{4}=(2,4,6,9) \quad \mathrm{M}=15
\end{array}
$$

(10 Marks)

